Homework 1

Chance and Uncertainty (PH3243)
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Problem 1. *Translate the sentence below into the language of propositional logic.*

"Bats are bugs or grass is green."

Problem 2. *Translate the sentence below into the language of propositional logic.*

"Addie and Anse will not both go to the party."

Problem 3. *Give the complete truth table for the sentence '$p \wedge q$'.*

Problem 4. *Give the complete truth table for the sentence '$p \vee (\neg q \rightarrow \neg p)$'.*

Problem 5. *Give the complete truth table for the sentence '$(p \vee \neg p) \wedge (q \leftrightarrow r)$'.*

Problem 6. *State the axioms for the version of probability theory that we are studying in this course (be sure to define what tautologies are, and what it is for two propositions to be mutually exclusive).*

Problem 7. *Let p be a proposition. Let Pr be a probability function such that $Pr(p) = .6$. What is $Pr(\neg p)$?*

Problem 8. *Let p and q be propositions, and let Pr be a probability function. Suppose $Pr(p \wedge q) = .3$, $Pr(p \wedge \neg q) = .2$, and $Pr(\neg p \wedge q) = .1$. What is $Pr(p)$?*

Problem 9. *Let p and q be propositions, and let Pr be a probability function. Suppose $Pr(p \vee \neg q) = .4$ and $Pr(\neg q \wedge \neg p) = .1$. What is $Pr(p)$?*

Problem 10. *In an intergalactic competition, the Yorks and the Zorks are competing to win the championship. Exactly one of these two teams will win. The probability that the Yorks win is $.75$. What is the probability that the Zorks win?*

Problem 11. *In an intergalactic competition, the Yorks and the Zorks are competing to win the championship. Exactly one of these two teams will win. The probability that the Zorks lose is $.16$. What is the probability that the Yorks win?*

Problem 12. *In an intergalactic competition, the Orks, the Borks, and the Corks are competing to with the championship. Exactly one of these three teams will win. The Orks and the Borks are equally likely to win, but the Corks are three times as likely to win as the Borks. How likely is it that the Orks will win? How likely is it that the Borks will win? And how likely is it that the Corks will win?*

Problem 13. *In an intergalactic competition, the Orks, the Borks, and the Corks are competing to with the championship. Exactly one of these three teams will win. The likelihood of the Borks losing is twice as great as the likelihood of the Orks winning. The Corks are twice as likely to win as the Borks. How likely is it that the Orks will win? How likely is it that the Borks will win? And how likely is it that the Corks will win?*

Problem 14. *In an intergalactic competition, the Alphas, the Betas, the Gammas, and the Deltas are competing to with the championship. Exactly one of these four teams will win. The Betas are half as likely to win as the Alphas. The likelihood that the Alphas win is twice as great as the likelihood that either the Gammas win or the Deltas win. The likelihood of the Deltas losing is four times as great as the likelihood of the Deltas winning. So how likely is it that the Alphas will win? How likely is it that the Betas will win? How likely is it that the Gammas will win? And how likely is it that the Deltas will win?*

Problem 15. *Let X and Y be logically equivalent propositions, and let Pr be a probability function. Prove that $Pr(X) = Pr(Y)$. Hint: start by showing that (i) $X \vee \neg Y$ is a tautology, and (ii) X and $\neg Y$ are mutually exclusive. Second hint: feel free to use—without proof—the fact that for any proposition Z, $Pr(Z) = 1 - Pr(\neg Z)$.*